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The study of emittance match condition in MICE lattice

D. Huang [a], D. Kaplan [a], T. Roberts [a, b], K. Tilley [c]
[a] *BCPS Department, Illinois Institute of Technology, Chicago, IL 60616, USA*
[b] *Muons Inc, Batavia, IL 60510, USA*
[c] *Rutherford Appleton Laboratory, Didcot, UK*

Strengths of the 6 quadrupoles and the diffuser thickness are found to obtain the desired emittance and match into MICE, under the assumption of an ideal Gaussian input beam. A further set of parameters are derived for a realistic muon beam obtained from pion decay, with TOF0 removed from the lattice. The features of the resulting beams are described. The single charged particle motion in a long solenoid is also studied to illustrate the physics picture of the emittance match into MICE.

1. INTRODUCTION

1.1. Ionization cooling and MICE

In order to achieve the desired muon collider luminosity or Neutrino Factory beam intensity, the 6-dimensional emittance of a muon beam must be reduced into an acceptable range; this process is called "cooling." Since a muon is very similar to an electron except that its mass is about 200 times larger, the best way to cool it is to use it to ionize some atoms, thereby decreasing its momentum.

To obtain good cooling efficiency, a low-Z material such as liquid hydrogen is used as the energy absorber. The basic formula of ionization cooling is (with the energy in GeV) [1] :

$$\frac{d\epsilon_{\rm n}}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_{\rm n}}{E_\mu} + \frac{1}{\beta^2}\frac{\beta_\perp (0.014)^2}{2E_\mu m_\mu L_{\rm R}} \tag{1.1}$$

where $\beta = v/c$, $\beta_\perp$ is the transverse betatron function at the absorber, $L_{\rm R}$ is the radiation length of the material, E_μ is the kinetic energy of a muon, m_μ is the mass of a muon, s is the longitudinal displacement of a muon in the absorber, and $\epsilon_{\rm n}$ is the normalized transverse emittance. The first term describes cooling effect due to energy loss; the second term describes heating due to multiple scattering.

MICE stands for Muon Ionization Cooling Experiment, which is being built at the Rutherford Appleton Laboratory (RAL) in the UK as a facility to demonstrate the ionization cooling scheme. Fig. 1 shows the layout of the MICE facility, and Fig. 2 illustrates the cooling section of MICE.

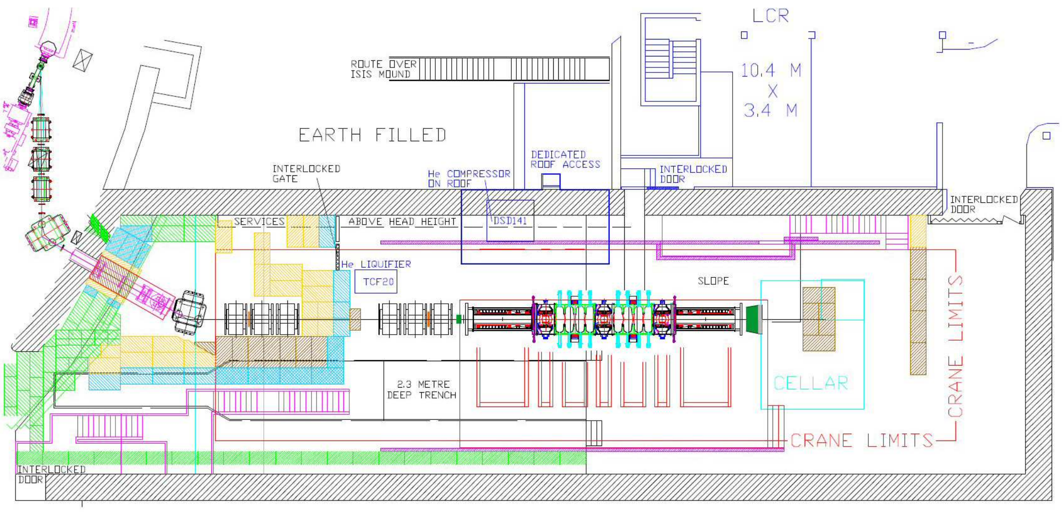


FIG. 1: The MICE layout at RAL

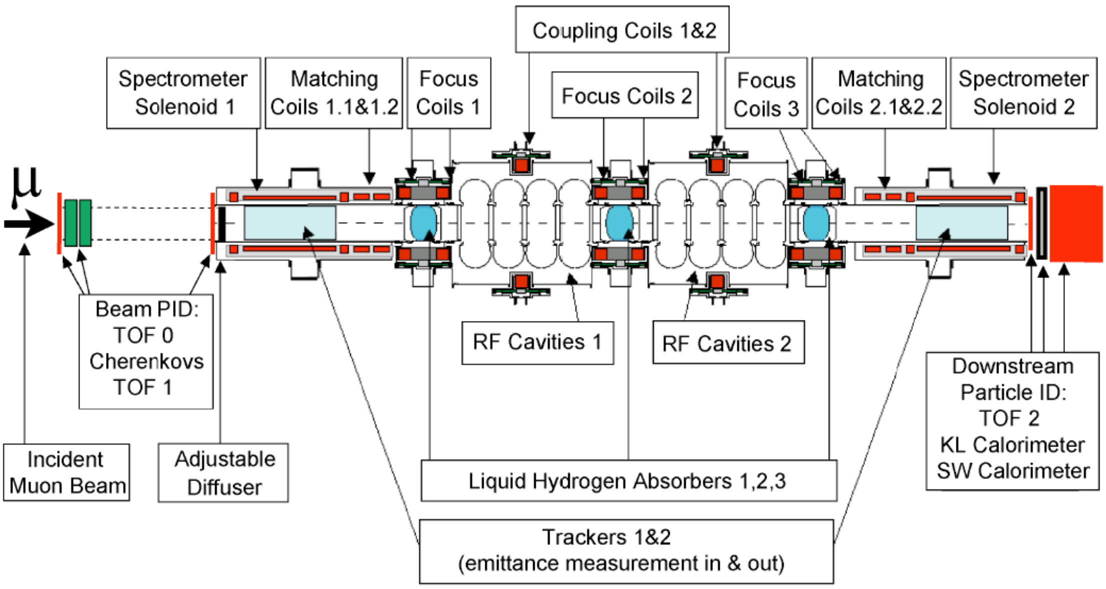


FIG. 2: The MICE cooling section

1.2. Tuning the emittance match of the MICE channel

At the upper-left corner of Fig. 1, the proton beam from the ISIS synchrotron hits the target to produce the Pion beam. Next to the target are there 3 quadrupoles Q1, Q2, and Q3, which is to focus beam into the decay solenoid. After these quadrupoles there is a bending magnet B1 to bend the Pion beam into the decay solenoid. Inside the decay solenoid, a small fraction of the Pions decay into muons. After the decay solenoid, there is another bending magnet B2 to bend the muon beam into a set of 6 quadrupoles, Q4→Q9, which are used (along with a lead diffuser with adjustable thickness before the first spectrometer solenoid) to match the beam emittance into the spectrometer solenoid. This section also includes a proton absorber, two Cherenkov counters and two planes of TOF counters. Following this is the "tracker 1" spectrometer solenoid, which is the beginning of the cooling section.

In order to test the reliability of the Monte Carlo, so that we can prove that ionization cooling is well understood, there are 3 desired transverse emittances of the muon beam at 3 different longitudinal momenta; moreover, the betatron function inside Tracker1 should be as constant as possible. If the emittance and betatron function of beam can reach those desired values, the beam is called "matched." In order to obtain a "matched" beam, one needs to understand the motion of beam inside the Tracker1 solenoid and to figure out what quadrupole strengths of Q4→Q9 and the thickness of the diffuser at the entrance of the Tracker1 solenoid should be used to obtain the best emittance match.

The important difference between the MuCool lattice we are discussing here and a regular accelerator lattice is: there is multiple scattering introduced by the particle IDs such as Time-Of-Flight detector, Cherenkov detector, etc. in our lattice. Therefore the reversibility of the optic system cannot be applied in our discussion. In order to obtain the best match, one must adjust the parameters such as the quadrupole strengths of Q4→Q9 and the diffuser thickness by hand.

The purpose of this report is to discuss the matching condition and method and the results of the emittance match.

2. CHARGED PARTICLE MOTION IN A SOLENOID

To understand the match condition, first we much investigate particle motion in the tracker1 solenoid. In my Ph.D. dissertation [2], I have studied such a topic in general.

In general, taking into account acceleration (although it does not matter in our discussion here), for a single charged particle inside a cylindrically symmetric, laminar, relativistic, coasting beam moving in a focusing channel of constant strength, following Newton's second law, we have the transverse equation of motion of a single particle:

$$r'' + \left(\frac{\gamma'}{\gamma\beta^2}\right) r' - \frac{F_{\text{ext}}}{\gamma m\beta^2 c^2} + \frac{r\dot{\theta}}{\beta^2 c^2} = 0 \tag{2.2}$$

where m is the rest mass of a charged particle, F_{ext} is the external force acting on the charged particle which can be from the solenoid focusing, rf focusing or the electro-magnetic field of other charged particles. (')

represents the longitudinal derivative $\frac{d}{ds}$, and r is the radial displacement of a single charged particle. $(\dot{})$ stands for the derivative to time t, *i.e.,* $\frac{d}{dt}$. The relationship between the two derivatives is

$$\frac{d}{ds} = \frac{d}{\beta c dt} \tag{2.3}$$

Usually, the last term in Eq. (2.2) can be assimilated into the external focusing due to the solenoid. To discuss it, first we introduce Busch's theorem.

The variation of the angular momentum of a single charged particle of charge e due to a magnetic field can be described by :

$$\frac{d\vec{L}}{dt} = m\frac{d}{dt}(\gamma\vec{r} \times \vec{v}) = e\vec{r} \times (\vec{v} \times \vec{B}) \tag{2.4}$$

or

$$m\frac{d}{dt}(\gamma r^2\dot{\theta})\hat{s} = e\vec{r} \times (\vec{v} \times \vec{B}) \tag{2.5}$$

Due to the fringe field of the solenoid, all the velocity components are introduced. Therefore

$$\vec{v} \times \vec{B} = (r\dot{\theta}B_S - B_\theta\dot{s})\hat{r} - (\dot{r}B_S - B_r\dot{s})\hat{\theta} + (B_\theta\dot{r} - B_r r\dot{\theta})\hat{s} \tag{2.6}$$

and there is no contribution to $\vec{L}$ in radial and longitudinal directions of $\vec{v} \times \vec{B}$, so we only consider the azimuthal component:

$$\vec{v} \times \vec{B} = (B_r\dot{s} - \dot{r}B_S)\hat{\theta} \tag{2.7}$$

and

$$B_r = -\frac{\partial A_\theta}{\partial s} \qquad B_S = \frac{\partial(rA_\theta)}{r\partial r} \tag{2.8}$$

Because in a long solenoid coil, the longitudinal component of the magnetic field is much larger than the other two components, we have $\vec{B} = \vec{\nabla} \times \vec{A} \approx \vec{B}_S$, then we can obtain

$$\frac{m}{r}\frac{d}{dt}(\gamma r^2\dot{\theta}) = -\frac{e\dot{r}}{r}\frac{d(rA)_\theta}{dr} = -\frac{e}{r}\frac{d}{dt}(rA_\theta) \tag{2.9}$$

Finally, we have

$$\Delta(\gamma r^2\dot{\theta}) = -\frac{e}{m}\Delta(rA_\theta) = -\frac{e}{m}\Delta(\frac{\Psi}{2\pi}) \tag{2.10}$$

where

$$\Psi = \int_0^r rdr\int_0^{2\pi} B_S d\theta \tag{2.11}$$

is the magnetic flux.

Eq. 2.10 can also be written as:

$$\Delta L = -e\Delta(\frac{\Psi}{2\pi}) \tag{2.12}$$

where again, L is the angular momentum of the particle, which is longitudinal.

The single charged particle motion in a long solenoid can be described as a longitudinal straight motion and a transverse circular motion with varying radius (which can further be decomposed into the radial and azimuthal motion). Based on Eq. 2.12, one can see that the transverse motion depends on the magnetic flux across the area enclosed by the transverse trajectory of the particle. Transversely, the particle will perform a betatron oscillation. From Eq. 2.12, we can see that when the magnetic flux Ψ decreases, which means the radius of the circular trajectory is reduced, the angular momentum L will increase, and vice versa. Also, if some conditions are satisfied such that the angular momentum $\vec{L}$ remains constant, the magnetic flux Ψ remains constant too. Inside a long solenoid, it means the radius of the transverse circular motion is a constant, therefore the Twiss parameter $\beta_{\perp}$ is a constant and so is the emittance. In this case the beam is called "match" to some certain emittance.

3. ACHIEVING THE EMITTANCE MATCH

In order to test the reliability of the Monte Carlo, some combinations of transverse emittance and muon momentum are desired. As already discussed, we are to adjust the lattice parameters to satisfy those requests. Table I shows a matrix of beam momenta and emittances representing the range of conditions over which it is desired to operate MICE. Our goal is to demonstrate how these conditions can be achieved while maintaining a well matched beam. In the left-most column, those are the desired muon momenta at the center of empty absorber1 (Fig. 2). The first row shows the desired transverse beam emittances right at the entrance of the tracker1 solenoid. Since in a solenoid, the horizontal and vertical motion of a particle is coupled, therefore here we use the concept of transverse emittance instead of single $\epsilon_{\mathrm{n,\ x}}$ or $\epsilon_{\mathrm{n,\ y}}$, and $\epsilon_{\mathrm{n,\ tran}} = \sqrt{\epsilon_{\mathrm{n,\ x}} \cdot \epsilon_{\mathrm{n,\ y}}}$. Note: all the emittances here are normalized.

	1	6	10
140	?	?	?
200	?	?	?
240	?	?	?

TABLE I: The desired transverse emittances at the entrance of tracker1 solenoid at different muon momenta measured at the center of empty absorber1, emittance in π·mm·rad, momentum in MeV/c.

The question-marks stand for the lattice parameters needed to obtain the desired emittances. Again, the work to be done is to tune 7 parameters, *i.e.,* the strengths of quadrupole magnets Q4, Q5, Q6, Q7, Q8, Q9 and the thickness of the diffuser at the front of the tracker1 solenoid.

Geant4-based beam tracking code G4beamline [3] developed by T. Roberts of Muons, Inc. is an useful tool to simulate particle tracks in MICE. It is more or less like a regular tracking code in accelerator physics except it is able to handle multiple scattering and energy loss. Moreover, to facilitate the tuning process, a Linux shell script called tunemice which is based on gminuit [4] employs the minuit minimizer to provide a friendly GUI to visualize the parameters. Fig. 3 illustrates the visual interface of tunemice shell script. With tunemice we are able to vary the 7 tuning parameters and see the beam profile and Twiss parameters visually. The tuning scheme can be simply described as the following: first we tune the quadrupole strength

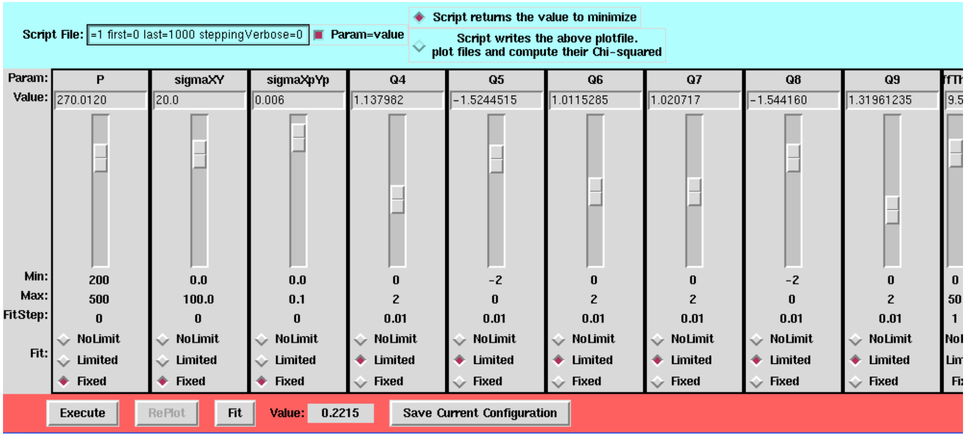


FIG. 3: The visual interface of tunemice shell script

$Q1$ with other parameters fixed to obtain the best match; then we tune the second parameter $Q2$ and so on until all the 7 parameters are tuned. This is the first round. Then we start the second round with the same steps beginning with tuning $Q1$. We keep doing these steps round by round until we find the final best match as we can (i.e., the emittance is closest to the desired one, and $\beta_\perp$ is nearly a constant.)

The tuning simulation starts just after the pion decay solenoid (before the proton absorber) and ends at the end of spectrometer solenoid 1 (tracker1 solenoid). It has been done in two ways: using an ideal Gaussian muon beam is created just before the proton absorber or using a more realistic beam obtained from the G4beamline simulation starting at the target. In the latter case, the 6-D phase space coordinates of every tracked particle just before the proton absorber are saved and recorded in a file that can be randomly sampled in the tuning optimization step (because sometimes there is no need to use all the saved particles for tuning optimization). In our study, the total number of particles to be sampled is 100,000. Taking into account the beam loss, it is statistically adequate. Since in the simulation, we find that the small number of outlier particles introduce considerable emittance distortion, a 3σ cut is applied to maintain reasonable emittance values.

For an ideal Gaussian beam, very good tuning results are obtained for emittances of 6π mm rad and 10πmm rad. However, it is very hard to obtain an emittance as low as 1π mm rad. The lowest emittance we can reach is $\approx 3\pi$ mm rad even without TOF0 which introduces considerable multiple scattering.

Figs. 4 and 5 show matching results for 6π mm rad and 10π mm rad emittance, respectively. The initial beam is Gaussian. The horizontal axis represents the original $\sigma_{\rm x}$ and $\sigma_{\rm y}$, which are the standard deviations in the horizontal and vertical directions and represent the transverse beam size.

The $\sigma_{\rm x'} = {\rm dx/dz}$ and $\sigma_{\rm y'} = {\rm dy/dz}$ are both 0.006, which represent the transverse beam spread. In practice the beam is approximately laminar and parallel, therefore we just introduce very small spread here.

Note that for different muon momenta, the emittance curves are all very close to the desired emittances no matter what the original beam size is, *i.e.,* 6π mm rad and 10π mm rad. Therefore we can conclude that; first, the tunes are all good for the 3 test muon momenta; second, the transverse emittance is very insensitive

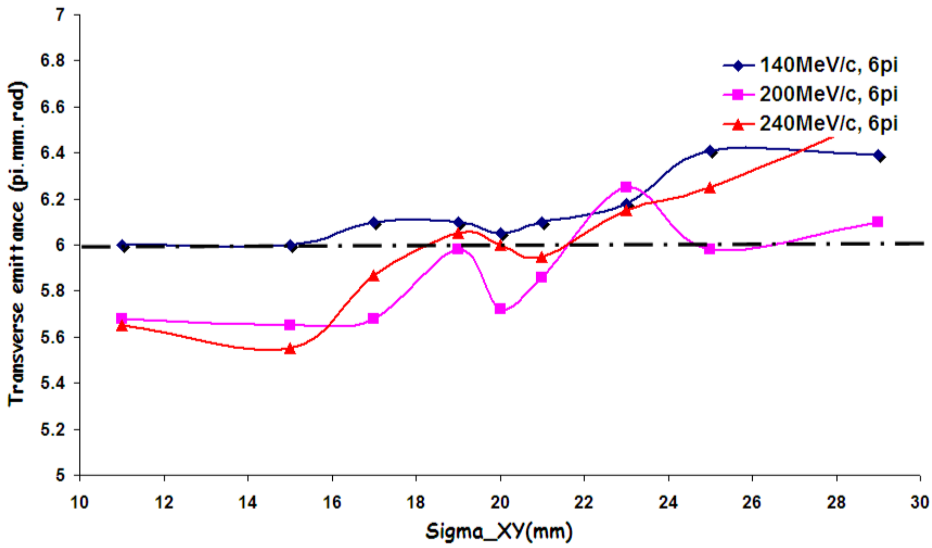


FIG. 4: The emittance matching curves at 6π mm rad. Different colors stand for different muon momenta at the center of the empty absorber1. The horizontal axis represents the standard deviation in horizontal and vertical directions, which are set equal to each other in the simulation. The Gaussian beam was used.

to the orginal beam size once the tune is fixed.

Fig. 6 illustrates the graphic output of tunemice for $6\,\pi$ mm rad emittance match at 200 MeV/c muon momentum. In the figure, we can see the emittance in tracker1 is very close to $6\,\pi$ mm rad and the variation of $\beta_\perp$ is from 260-400mm, which is not big and acceptable.

TABLE II shows the list of parameters with which we obtain $6\,\pi$ mm rad transverse emittance match for muon momenta of 140, 200 and 240MeV/c at the center of empty absorber1 for a Gaussian beam, where the quadrupole strength is in tesla/m.

p_μ(MeV/c)	$\sigma_{x,y}$(mm)	$\sigma_{x',y'}$	Q4	Q5	Q6	Q7	Q8	Q9	Diffuser Thickness(mm)
140	20	0.006	0.88614	-1.18698	0.80519	0.81250	-1.22917	1.05043	9.5
200	20	0.006	1.13798	-1.52445	1.01153	1.02072	-1.54416	1.13961	9.5
240	20	0.006	1.28164	-1.71672	1.13910	1.49452	-1.73692	1.48604	9.5

TABLE II: The parameters which are used to obtain 6π mm rad transverse emittance match at a variety of muon momenta. The quadrupole strengths are in tesla/m. The Gaussian beam was used. The diffuser thickness was fixed at 9.5mm.

The ideal Gaussian beam shows very good matching results. However, for the "realistic" μ^+ beam obtained from the decay of π^+'s produced in the target, we are not able to obtain matching results as good for a Gaussian beam. For example, in order to obtain 6π mm rad transverse emittance at the entrance of the tracker1 solenoid, we have to remove TOF0. Although the match is not very ideal, we are able to obtain a factor of 2 more "good" μ^+ than before. *i.e.,* 1417 events/s.

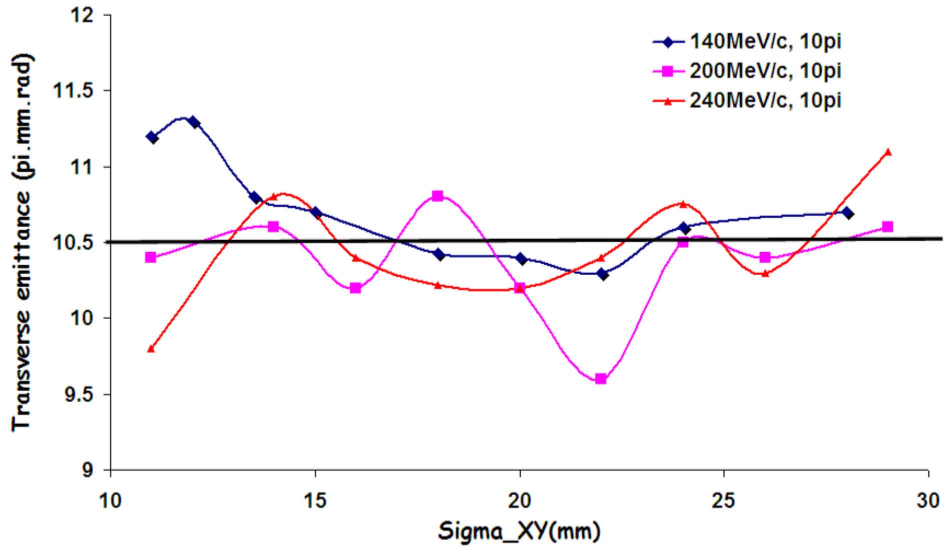


FIG. 5: The emittance matching curves at 10π mm rad. Different colors stand for different muon momenta at the center of the empty absorber1. Horizontal axis represents the standard deviation in horizontal and vertical direction, which are set equal to each other in the simulation. The Gaussian beam was used.

Fig. 7 shows the graphic output 6π mm rad emittance match for a "realistic" beam. The meaning of each plot are identical to those in Fig. 6. Note that the TOF0 is removed.

TABLE III illustrates the parameters of the "realistic" emittance match. The transverse emittance is 6π mm rad and the muon momentum is 200MeV/c.

P_μ(MeV/c)	Q4	Q5	Q6	Q7	Q8	Q9	Diffuser Thickness(mm)
200	1.19488	-1.60067	1.06211	1.07175	-1.62137	1.19659	3

TABLE III: The parameters which are used to obtain 6π mm rad transverse emittance match at variety of muon momenta. The quadrupole strength is in tesla/m. Note that beam is obtained from pion decay, not Gaussian any more.

In Fig. 7 we see that the variation of $\beta_\perp$ in the tracker1 solenoid is from 250mm-650mm, which seems too large. That is because of the reduction of the diffuser thickness. The diffuser thickness was reduced because for 200 MeV/c muon momentum, we are not able to obtain $6\pi\, mm\, rad$ emittance with the diffuser thickness of 9.5 mm. As we have discussed in Sec. 2, the diffuser thickness is varied to adjust the angular momentum in order to satisfy the match condition, and in the ideal match, $\beta_\perp$ is a constant. If the diffuser is too thin, it will not perform this function very well. Thus the large variation of $\beta_\perp$ is not surprising, since we have reduced the diffuser thickness to 3 mm.

In addition, we studied the effect of the diffuser thickness. Keeping the rest of parameters fixed, we vary the diffuser thickness to see its influence on the transverse emittance and betatron function. Fig. 8 shows how the transverse emittance and beta function vary with diffuser thickness. In Fig. 8, we can also see that $\beta_\perp$ is approaching a constant for diffuser thickness greater than 15mm, so is the transverse emittance.

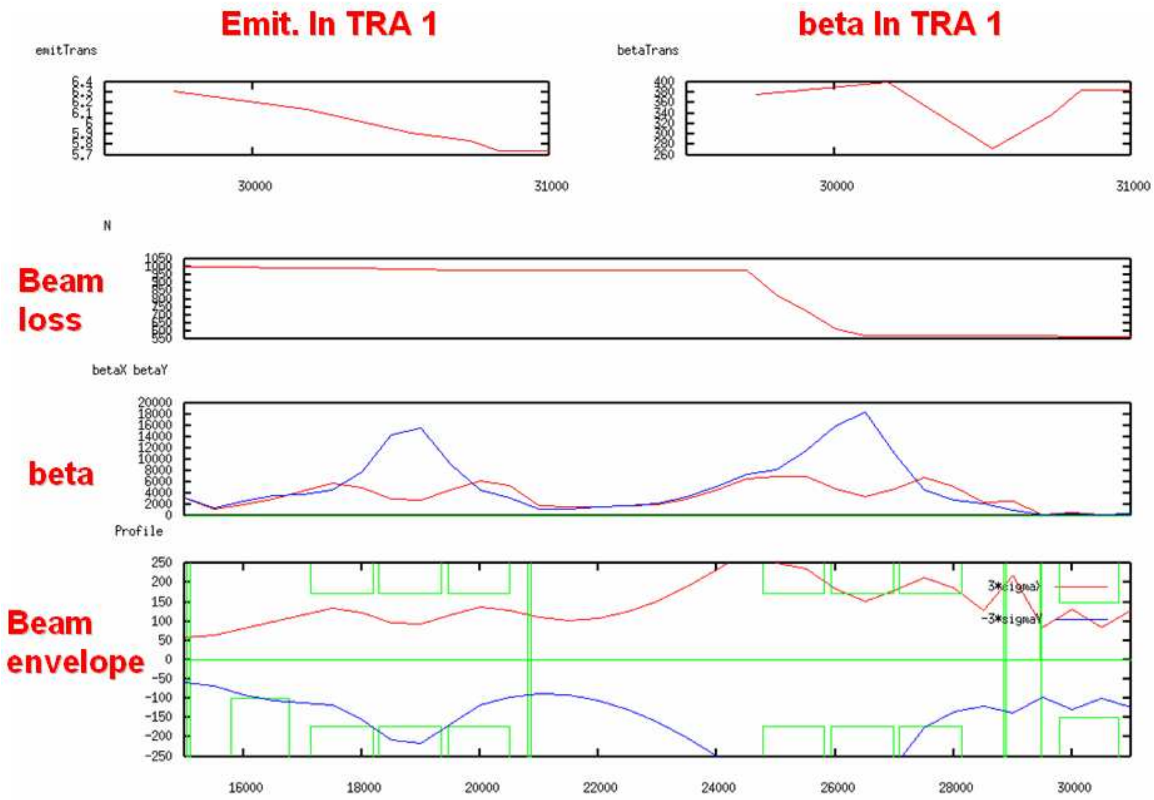


FIG. 6: The graphic output for 6π mm rad transverse emittance match at 200MeV/c muon momentum for a Gaussian beam. All the horizontal axises stand for the longitudinal coordinates. The top two plots are the emittance curve and $\beta_\perp$ curve in solenoid tracker 1, from left to right; the bottom three plots are the beam loss, $\beta_\perp$, and beam profile curves from the entrance of proton absorber to the exit of solenoid tracker 1.

This is probably because of the scraping introduced by the limitation of solenoid tracker 1.

4. CONCLUSIONS

The desired normalized transverse beam emittance in MICE can be achieved by adjusting the quadruple strengths and diffuser thickness in the matching section. Very good match results are obtained for a Gaussian beam. The match for a "realistic" μ^+ beam obtained from π^+ decay is not very ideal. However, in the best case, the lattice parameters can be set to obtain a factor of 2 more "Good" μ^+ particles than before.

G4Beamline is a very good tool to simulate the beam optics with multiple scattering. In the simulation, small amount of outlier particles introduce a huge emittance distortion. Therefore use "σ"-cut is necessary to obtain the reasonable emittance.

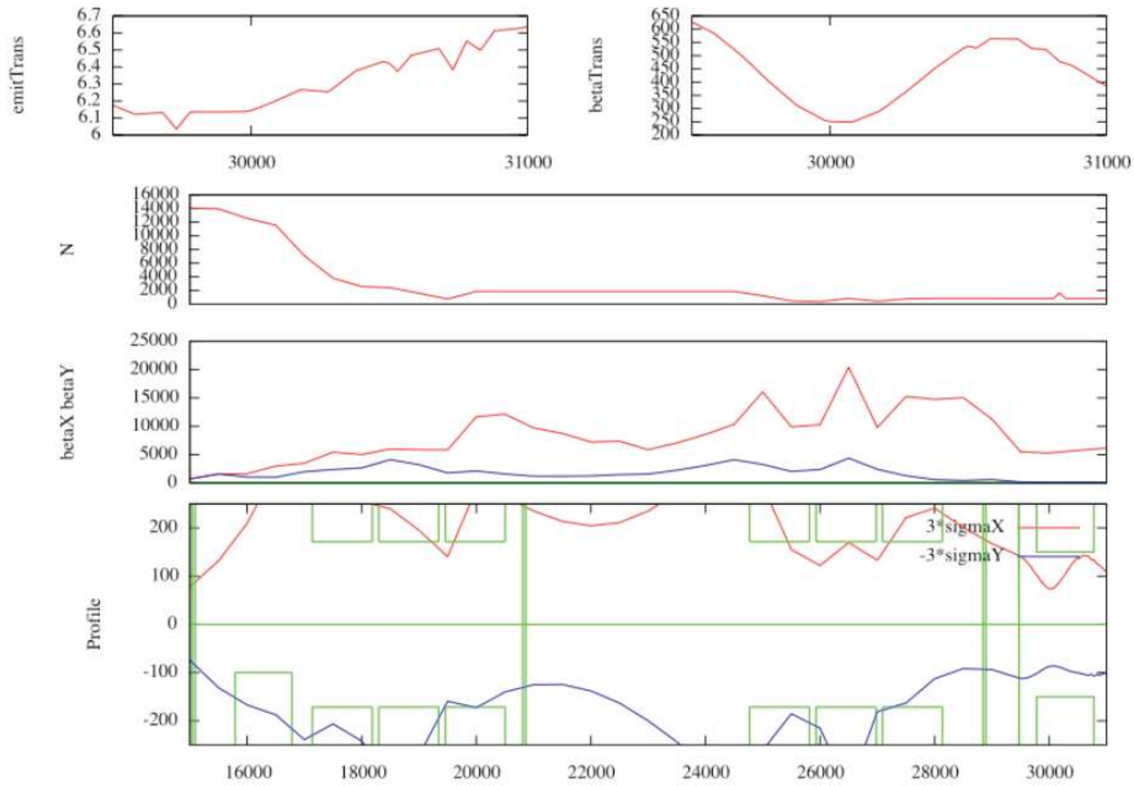
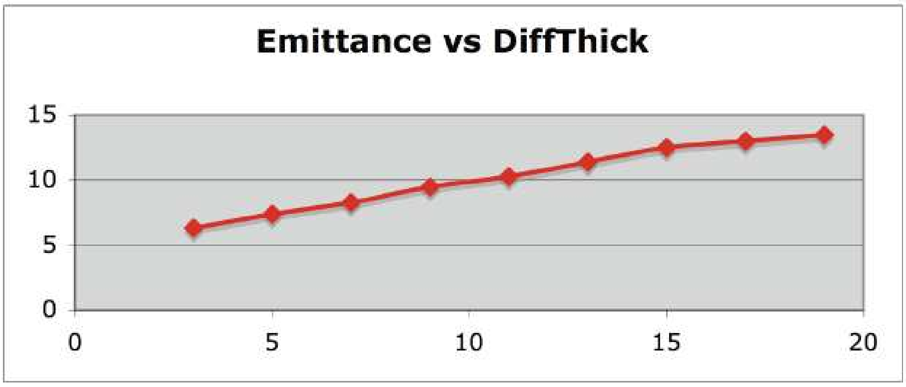


FIG. 7: The graphic output for 6π mm rad transverse emittance match with 200MeV/c muon momentum for a "realistic" μ^+ beam obtained from π^+ decay. TOF0 is removed. All the horizontal axises stand for the longitudinal coordinates. The top two plots are the emittance curve and $\beta_\perp$ curve in solenoid tracker 1, from left to right; the bottom three plots are the beam loss, $\beta_\perp$, and beam profile curves from the entrance of proton absorber to the exit of solenoid tracker 1.

In order to obtain the desired transverse emittance, the diffuser is indispensable. Its thickness plays a very important role because it strongly influences the angular momentum of a charged particle, which is a key factor to determine the betatron motion of a charged particle in a solenoid.

The parameters for the best emittance match have been obtained, and to be tested and confirmed in the actual MICE lattice.



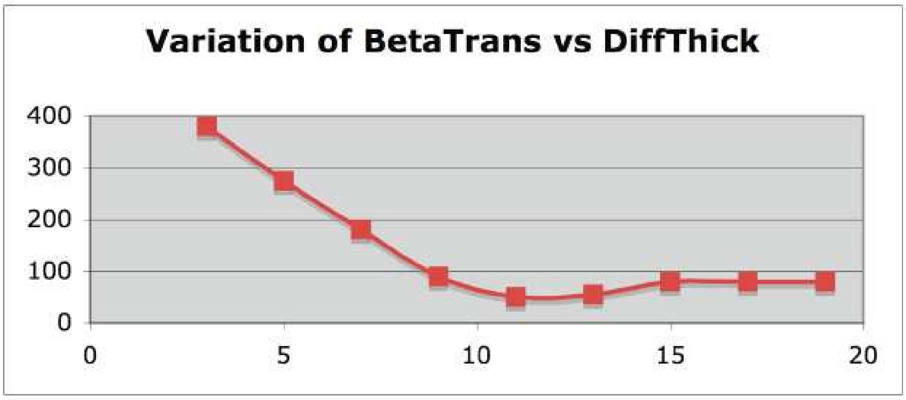


FIG. 8: The normalized transverse emittance and $\beta_\perp$ inside the tracker1 solenoid as a function of diffuser thickness.

Acknowledgments

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[2] Dazhang Huang, Ph.D. dissertation, unpublished, 2006.
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[4] Graphic Interface software, Muons. Inc. see http://www.muonsinc.com/